



가 가

- Ayache-Forsyth-Vetzal (2003) -

1.			
1.1			
1.2			
2.			
3.	가		
3.1	가		
3.2		가	
3.2.1	Ingersoll (1977)		
3.2.2	Brennan-Schwartz (1977)		
3.2.3	Fisher (1978)		
3.2.4	Goldman-Sachs (1994)		
3.2.5	Tsiveriotis-Ernandes (1998)		
4.			
4.1	Ayache-Forsyth-Vetzal (2003)		
4.1.1	(Credit Risk)		가
4.1.2	(Credit Risk)		가
4.1.3			
4.2	(Parameter)		
4.3			
4.4	(Sensitivity Analysis)		

B : bond component, B_c : call price, B_p : put price, q : dividend rate

$$B_t + SB_t + \frac{1}{2} S^2 B_{ss} - (r + s)B = 0 \quad (3-5)$$

Tsiveriotis-Fernandes (3-4) (linear complementarity problem)

$$V = V - \left(\frac{1}{2} S^2 V_{ss} + (q - r)SV_s - (V - B) - (r + s)B\right) \quad (3-6)$$

, = T-t

B_c S B_p S 가 V = B + C [B : bond component, C: equity component]

cash-only part가 (equity component) credit risk

가 Tsiveriotis-Fernandes 가 2 가

가

4. 1 Ayache-Forsyth-Vetzal (2003)

가 Black-Scholes 가 (the value of the issuing firm)

(the basic underlying state variable) 가(the issuing firm's

stock price) (the basic underlying factor)

가 Ingersoll(1977), Brennan and Schwartz(1977, 1980), Merton(1974),

Black and Cox(1976), Longstaff and Schwartz(1995), Nyborg(1996) 가

(debt) (equity) 가

가

Fisher(1978), McConnell and Schwartz(1986), Goldman-Sachs(1994), Tsiveriotis-

Fernandes(1998), Yigitbasioglu(2001) 가

가 가

10) Ayache, E., Forsyth, P. A. and Vetzal, K. R., " Valuation of Convertible Bonds With Credit Risk ", The Journal of Derivatives, Fall, 2003, pp 9~ 29

approach) Jarrow and Turnbull(1995), Duffie and Singleton(1999), Madan and Unal(2000) (the reduced form (risky debt) 가

(single jump loss) 가 가 (probability

of default) (hazard rate) 가

가 Tsiveriotis-Fernandes (1998), Davis and Lischka(1999), Takahashi-Kobayashi-Nakagawa(2001), Hung and Wang(2002), Anderson and Buffum(2003) Tsiveriotis-Fernandes (1998) 가

approach) 가 (the reduced form

가 Clark and Weinstein(1983) 가 30%

가 Beneish and Press(1995) 가 300 62% 가

가 가

가 가

가 Ayache-Forsyth-Vetzal 가 0% 100% (general-factor framework) 가 Poisson default process 가 (linear complementarity problem)

(credit risk) 가 가

가 (default risk) 가

가 가

가 가

4. 1. 1 (Credit Risk) 가

dS = μSdt + Sdz ----- (4-1) 가

가 V(S, t) ,

$$V_t + \left(\frac{1}{2}\sigma^2 V_{ss} + (r(t) - q)SV_s - (r(t) - q)V\right) = 0 \quad (4-2)$$

, $r(t)$: q :
(4-2)

$$V = -V_t - \left(\frac{1}{2}\sigma^2 V_{ss} + (r(t) - q)SV_s - (r(t) - q)V\right) \quad (4-3)$$

(linear complementarity problem)

가

4. 1. 2 (Credit Risk) 가

Call option 가

$$S^* = S(1 - \dots) \quad (4-4), 0 \leq 1$$

(the total default) $= 1$
(the partial default) $= 0$

$$= V - S \quad (4-5)$$

(credit risk)가, $p=0$

$$d = \left[V_t + \frac{\sigma^2}{2} V_{ss} \right] dt + o(dt) \quad (4-6)$$

(hazard rate) p 가 가, 가 (4-4)

$RX(0, R, 1, R : X : 가$

가 가 가 $S(1 -)$ 가, t 가 $t+dt$

(d)

$$d = (1 - pdt) \left[V_t + \frac{\sigma^2}{2} V_{ss} \right] dt - pdt(V - S) + pdt \max[S(1 -), RX] + o(dt)$$

$$= \left[V_t + \frac{\sigma^2}{2} V_{ss} \right] dt - pdt(V - V_s S) + pdt \max[S(1 -), RX] + o(dt) \quad (4-7)$$

(4-7) ,
[V - SV]dt

$$= \left[V_t + \frac{\sigma^2}{2} V_{ss} \right] dt - p(V - V_s S) dt + p \max[S(1 -), RX] dt + o(dt) \quad (4-8)$$

(4-8)

$$V_t + \left[(r(t) + p)SV_s + \frac{\sigma^2}{2} V_{ss} - (r(t) + p)V + p \max[S(1 -), RX] \right] = 0 \quad (4-9)$$

(4-9) (t) drift term (t)+p discounting term, $R=0$
 $= 1$ (the total default)

가 (4-3) (4-3) (t) (t)+p
Takahashi-Kobayashi-Nakagawa(2001) Anderson-Buffum(2003)

$$MV - V_t - \left[\frac{1}{2}\sigma^2 V_{ss} + (r(t) + p - q)SV_s - (r(t) + p)V \right] = 0 \quad (4-10)$$

(4-9) (4-10)

$$MV - p \max[S(1 -), RX] = 0 \quad (4-11)$$

(4-11)

(linear complementarity problem)

$$B_c > S, [MV - p \max[S(1 -), RX] = 0, (V - \max(B_p, S) = 0, (V - B_c) = 0]$$

$$[MV - p \max[S(1 -), RX] = 0, (V - \max(B_p, S) = 0, (V - B_c) = 0]$$

$$[MV - p \max[S(1 -), RX] = 0, (V - \max(B_p, S) = 0, (V - B_c) = 0] \quad (4-12)$$

$$B_c < S, V = S \quad (4-13)$$

(4-12) (4-13)

가 (V)

$$MV - p \max[S(1 -), RX] = 0, \quad (4-14)$$

$\max(B_c, S)$

(4-12) (4-13)

risk

가 Brownian motion

1) Takahashi, A., Kobayashi, T. and Nakagawa, " Pricing Convertible Bonds with Default Risk ", The Journal of Fixed Income, Dec, 2001, pp 20-29

12) Anderson, L. and Buffum, D, " Calibration and Implementation of Convertible Bond Model ", Working paper, Bank of America Securities, 2003

4. 1. 3

Ayache-Forsyth-Vetzal Credit Risk
 (Finite Difference Method) 가
 Tsiveriotis-Fernandes (1998)
 (Explicit Finite Difference Method)

$$\frac{V}{t} + (S, t) \frac{\partial^2 V}{\partial S^2} + b(S, t) \frac{\partial V}{\partial S} + c(S, t) = 0 \quad (4-15)$$

가 T, 가 S_{max}, 가 0, 가 S_{max}, 가
 T, K, S_{max}, I, (Grid)
 t = T/K, s = S_{max}/I, Grid (i · S, T - k · t)

$$V_i^k = V(i \cdot s, T - k \cdot t) \quad (4-16)$$

, Taylor approximation

$$\frac{\partial V}{\partial t}(S, t) = \frac{V_i^k - V_i^{k+1}}{t} + O(t)$$

$$\frac{\partial V}{\partial S}(S, t) = \frac{V_{i+1}^k - V_{i-1}^k}{2S} + O(S^2)$$

$$\frac{\partial^2 V}{\partial S^2}(S, t) = \frac{V_{i+1}^k - 2V_i^k + V_{i-1}^k}{S^2} + O(S^2) \quad (4-17)$$

(4-15) (approximation formula)

$$\frac{V_i^k - V_i^{k+1}}{t} + a_i^k \left[\frac{V_{i+1}^k - 2V_i^k + V_{i-1}^k}{S^2} \right] + b_i^k \left[\frac{V_{i+1}^k - V_{i-1}^k}{2S} \right] + c_i^k V_i^k = 0 \quad (4-18)$$

(Error Term)

$$V_i^k - V_i^{k+1} + a_i^k (V_{i+1}^k - 2V_i^k + V_{i-1}^k) + b_i^k \left(\frac{V_{i+1}^k - V_{i-1}^k}{2} \right) + c_i^k V_i^k = 0 \quad (4-19)$$

(, V₁ - $\frac{t}{S^2}$, V₂ - $\frac{t}{S}$)

< -1> 13

					가	
	2001. 4. 9	2003.10. 9	1%	8%	3,277	BBB
	2001. 6.13	2006. 6.13	1%	2%	5,000	BB-
	2001. 6.29	2004. 6.29	2%	8%	30,000	BB
	2001.12.10	2004. 3.31	1%	10%	5,400	BB +
	2001.12.24	2004.12.24	1%	8%	5,000	BBB-
	2002. 3. 5	2004.11.30	1%	6.5%	35,000	BB +
(15)	2002. 4.12	2007. 4.12	0%	0%	5,000	BB-
	2002. 4.19	2004.12.31	1%	7.5%	9,720	BB +
	2002. 7.12	2005. 7.12	5%	8.2%	5,000	BBB-
	2002.10.24	2005.12.31	1%	6.3%	5,000	BBB +
(16)	2003. 2.10	2008. 2.10	0%	0%	5,000	BB-
	2003. 7.11	2006. 7.11	4%	8%	12,450	BBB
LG	2003. 7.21	2009. 1.21	3%	8%	21,500	A +

$$V_i^{k+1} = (a_i^k V_i^k - \frac{b_i^k V_2}{2}) V_{i-1}^k + (1 - 2a_i^k + tC_i^k) V_i^k \quad (4-20)$$

가

(4-3)

(4-10), = T-t

$$V = V - \left(\frac{1}{2} \right) S^2 V_{ss} + ((t) - q) S V_s - r(t) V \quad (4-21)$$

$$MV = V - \left[\frac{1}{2} S^2 V_{ss} + ((t) + p - q) S V_s - (t) + p \right] V = 0 \quad (4-22)$$

가 (V), Tsiveriotis-Fernandes (3-6)

4. 2 (Parameter)

BB- 13

2001

spot rate 가



< -2> 가 ()

	가 ()	가 ()	가()	(%)
	3,570	400	3,170	0.8880
GPS	625	140	485	0.7760
	1,140	320	820	0.7193
	11,190	3,850	8,050	0.6765
	1,050	810	240	0.2286
	480	105	375	0.7813
	800	140	660	0.8250
	85	60	25	0.2941
	1,220	255	965	0.7910
	335	285	50	0.1493
	6,470	1,320	5,150	0.7960
	490	90	400	0.8163
	545	105	440	0.8073
	375	140	235	0.6267
	3,105	660	2,445	0.7874
	640	340	300	0.4688
	500	250	250	0.5000
	820	400	420	0.5122
	370	115	255	0.6892
	35	20	15	0.4286
	130	100	30	0.2308
	200	200	0	0
	225	130	95	0.4222
	255	115	140	0.5490
	190	75	115	0.6053
				=0.5747

* 2000 Stock Database 25

가() 가 Stock Database
 가 가 가 10,000
 가 가 10 가

< -3> 가

	가	가	가	가	가	
	2001. 4. 9	1,655	1.1237%	6.75%	0.80	3.052
	2001. 6.13	1,135	2.2935%	7.06%	0.87	2.000
	2001. 6.29	2,320	0.0%	6.21%	0.98	0.333
	2001.12.10	5,380	1.9999%	6.22%	0.80	1.852
	2001.12.24	3,780	1.7250%	5.14%	0.80	2.000
	2002. 3. 5	35,500	0.6678%	6.08%	0.73	0.286
(15)	2002. 4.12	1,320	1.7152%	7.19%	1.01	2.000
	2002. 4.19	8,810	0.5982%	6.59%	0.90	1.029
	2002. 7.12	3,365	3.2951%	6.09%	0.66	2.000
	2002.10.24	3,250	1.1336%	5.54%	0.76	2.000
(16)	2003. 2.10	2,305	1.4297%	4.94%	1.97	2.000
	2003. 7.11	11,950	0.1699%	4.51%	0.56	0.803
LG	2003. 7.21	18,700	0.0%	4.96%	0.75	0.465

* (Parameter) 56-57p

가 Historical Volatility
 가 가
 (Probability of default) Jarrow-Turnbull
 (Recovery rate) Moody's Special Comment "Default & Recovery Rates of Corporate Bond Issuers(2003)
 가 2000 가 25
 가 (4-2) 13 가 (= 0.5747)
 가 (4-3)

4.3

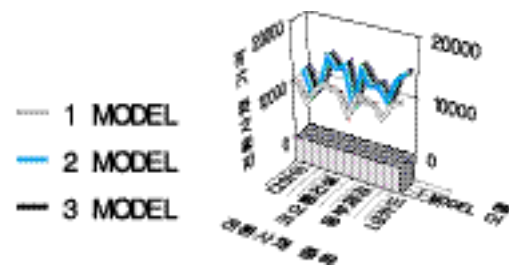
가 (-4) (-1) .

< -4> 가

	Ayache-Forsyth-Vetzal Model (Credit Risk)	Ayache-Forsyth-Vetzal Model (No Credit Risk)	Tsiveriotis-Fernandes Model
	9,415	11,003	10,903
	6,406	7,073	7,123
	9,100	9,199	9,195
	10,399	14,459	14,143
	9,986	12,702	12,470
	9,719	13,668	13,461
(15)	5,951	6,864	6,921
	9,875	13,752	13,646
	9,396	11,303	11,198
	9,289	11,557	11,501
(16)	7,238	9,517	8,643
	9,882	13,364	13,182
LG	10,796	14,766	14,650

* (Credit Risk) Ayache-Forsyth-Vetzal Model, Model, Tsiveriotis-Fernandes Model 가 Ayache-Forsyth-Vetzal Model 가

< -1> 3 MODEL 가



- 1 MODEL : Ayache-Forsyth-Vetzal Model (Credit Risk)
- 2 MODEL : Ayache-Forsyth-Vetzal Model (No Credit Risk)
- 3 MODEL : Tsiveriotis-Fernandes Model

13 가 (4-4) (4-1) , 13
 (Credit Risk) Ayache-Forsyth-Vetzal Model 가
 Ayache-Forsyth-Vetzal Model 가 가
 가 , Tsiveriotis-Fernandes Model 가
 Ayache-Forsyth-Vetzal Model 가 , Tsiveriotis-Fernandes Model
 Ayache-Forsyth-Vetzal Model “0”
 (Partial Default) 가 ,
 Ayache-Forsyth-Vetzal Model “0” 가 Tsiveriotis-Fernandes Model
 Model 가
 13 (15) Tsiveriotis-Fernandes Model
 가 Ayache-Forsyth-Vetzal Model 가 가
 5 2% (15) 0%
 가
 (Credit Risk) Ayache-Forsyth-Vetzal Model 가 가
 10,000 가 LG (10,796) (10,399) 2 , 11
 8 가 가 가 , (15) ,
 (16) , 3 가

4.4 (Sensitivity Analysis)

가 가 가
 가 가
 (q), (R_i) 가 (), 가 ()⁶ (Parameter) 10% 가
 (Credit Risk) Ayache-Forsyth-Vetzal Model 가

< -5> (9,415)

			가	
(q)	1.1237%	1.2361%	9,414	-0.01%
(R _i)	6.75%	7.43%	9,262	-1.63%
가 ()	0.80	0.88	9,528	+1.20%
가 ()	0.5747	0.6270	9,278	-1.46%

* 10% 가 가

< -6> (10,399)

			가	
(q)	1.9999%	2.1999%	10,393	-0.06%
(R _i)	6.22%	6.84%	10,271	-1.23%
가 ()	0.80	0.88	10,565	+1.60%
가 ()	0.5747	0.6270	10,052	-3.34%

< -7> (9,986)

			가	
(q)	1.7250%	1.8975%	9,980	-0.06%
(R _i)	5.14%	5.65%	9,854	-1.32%
가 ()	0.80	0.88	10,140	+3.94%
가 ()	0.5747	0.6270	9,756	-2.43%

(4-5) (4-10) 6 (q) 10% 가
 가 1.14% 1.63% 가 (R_i) 10% 가
 가 1.46% 3.54% 가 () 10% 가
 (q), (R_i), 가 () 가
 가 () 1.20% 3.94% 가

< -8> (9,875)

			가	
(q)	0.5982%	0.6580%	9,872	-0.03%
(R _i)	6.59%	7.25%	9,732	-1.45%
가 ()	0.90	0.99	10,037	+1.64%
가 ()	0.5747	0.6270	9,575	-3.54%

< -9> (9,289)

			가	
(q)	1.1336%	1.2470%	9,286	-0.03%
(R _i)	5.54%	6.09%	9,144	-1.56%
가 ()	0.76	0.84	9,429	+1.51%
가 ()	0.5747	0.6270	9,087	-2.17%

< -10> (9,882)

			가	
(q)	0.1699%	0.1869%	9,881	-0.01%
(R _i)	4.51%	4.96%	9,769	-1.14%
가 ()	0.56	0.62	10,051	+1.71%
가 ()	0.5747	0.6270	9,574	-3.12%

(Convertible Bond) (Hybrid Security)
 가 가 가 가 가 가
 가 가 Black-Scholes (1973) 가 가
 가 가 가 가 가 가
 Approach) 가 가 가 가 가 가
 가 가 가 가 가 가
 Ayache-Forsyth-Vetzal (2003) 가 0% 100% 가
 가 가 (credit risk) 가

가 , 가 , ,
 (Credit Risk) Ayache-Forsyth-Vetzal Model Ayache-
 Forsyth-Vetzal Model Tsiveriotis-Fernandes Model 가
 가 가 () 가 가 ()
 가 가 "0" (Partial Default)
 Ayache-Forsyth-Vetzal Model 가 Tsiveriotis-Fernandes Model 가
 Ayache-Forsyth-Vetzal Model 가 () 가
 가 () 가 가 가

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